The author wishes to express his gratitude to Dr. R. G. Sturm, member of the Design Division of the Pressure Vessel Research Committee of the Welding Research Council and former Director of the Auburn Research Foundation and Engineering Experiment Station, for his advice and helpful suggestions; to Mr. Andrew King, former Laboratory Mechanician of the Department of Mechanical Engineering at Auburn University, for his help in the construction of the testing facilities. The research project has been sponsored by the Pressure Vessel Research Committee of the Welding Research Council through the Auburn Research Foundation and the Engineering Experiment Station of Auburn University. The author is grateful to the Pressure Vessel Research Committee Design Division's Subcommittee on Heads Convex to Spherical heads for the testing facilities. The research project has been available.

Table 1 gives the observed collapse pressure, the ASME Boiler and Pressure Vessel Code allowable pressure, and the ratio of collapse pressure to code pressure for the four tests. It should be noted that only the values given for the torispherical and ellipsoidal heads applied to heads. The other two, namely, those for the 120-deg and 90-deg toriconical heads, represent the collapse pressure for the cylindrical shells. The lengths of 34.7 in. and 38.4 in. given by the author are the lengths specified by the Code in Par. UG-28 for determining the allowable pressure for cylindrical shells.

Using the above lengths in the equation on which the Code charts are based and applying a factor of 4 against collapse, the writer obtains a Code allowable pressure of 3.9 psi for the 120-deg head and 5.3 psi for the 90-deg head. The corresponding ratios are 3.7 and 4.5.

Figs. 27 and 37 indicate that a toriconical head serves effectively as a stiffening ring so that it should not be necessary to add one third of the depth of such a head in determining the value of \(L/D_o\) for the shell. Using a value of \(L = 30\) in. in the Code analysis, the Code allowable pressure is found to be 6.8 psi for both vessels. This gives pressure ratios of 3.2 for the vessel with the 120-deg conical head and 3.5 for the vessel with the 90-deg conical head.

All these ratios of collapse pressure to Code allowable pressure are less than the theoretical factor of 4 against collapse for perfect vessels on which the Code charts are based. However, the out-of-roundness tolerances permitted by the code may reduce the ultimate pressure to 80 per cent of that of the corresponding perfect shell. This means that the factor against collapse may be as low as 3.2. The results obtained for the four vessels as modified above by taking the shell length only for the vessels with toriconical heads are comparable with this.

The Code rule for calculating the allowable pressure on the two toriconical heads is given in Par. UG-33(f) (2). If the change in angle can be considered to be the equivalent of a stiffening ring, the Code pressure is 16.8 psi for the 120-deg head and 10.3 psi for the 90-deg head. These values are greater than those found for the adjacent shell. If the inside diameter of the cone section is taken as the length \(L\), then the calculated value of the Code pressure is 3.4 psi. This would indicate that the heads were weaker than the adjacent shell, which was not the case.

The Code rules need to be reconsidered in the light of the results obtained from these tests. It is hoped that further tests will be of great value in this respect. A greater volume of data is needed in order to determine the strength of heads under external pressure.

\[ \text{DISCUSSION} \]

Elmer O. Bergman

The author's paper is a valuable contribution to the literature on vessels under external pressure. It furnishes test data on formed heads under external pressure which up to now have not been available.

Figs. 26, 27, 36, and 37 show the appearance of the four heads tested after collapse. The pictures reveal that collapse of the torispherical and ellipsoidal heads occurred in the head with only secondary distortion of the cylindrical shell. The reverse seems to be true of the two toriconical heads. Here the collapse is very great in the shells with relatively minor distortion of the heads. Thus it would appear that the tests do not give a true measure of the strength of toriconical heads under external pressure.

Table 1 gives the observed collapse pressure, the ASME Boiler and Pressure Vessel Code allowable pressure, and the ratio of collapse pressure to code pressure for the four tests. It should be noted that only the values given for the torispherical and ellipsoidal heads applied to heads. The other two, namely, those for the 120-deg and 90-deg toriconical heads, represent the collapse pressure for the cylindrical shells. The lengths of 34.7 in. and 38.4 in. given by the author are the lengths specified by the Code in Par. UG-28 for determining the allowable pressure for cylindrical shells.

Using the above lengths in the equation on which the Code charts are based and applying a factor of 4 against collapse, the writer obtains a Code allowable pressure of 3.9 psi for the 120-deg head and 5.3 psi for the 90-deg head. The corresponding ratios are 3.7 and 4.5.

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All these ratios of collapse pressure to Code allowable pressure are less than the theoretical factor of 4 against collapse for perfect vessels on which the Code charts are based. However, the out-of-roundness tolerances permitted by the code may reduce the ultimate pressure to 80 per cent of that of the corresponding perfect shell. This means that the factor against collapse may be as low as 3.2. The results obtained for the four vessels as modified above by taking the shell length only for the vessels with toriconical heads are comparable with this.

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The Code rules need to be reconsidered in the light of the results obtained from these tests. It is hoped that further tests will be of great value in this respect. A greater volume of data is needed in order to determine the strength of heads under external pressure.
be made in which the thickness of the cones is such that they will collapse before the shell does.

F. V. Hartman

The author is to be complimented on his excellent and complete description of the test specimens, the method and procedure of testing, and the behavior of the specimens under test. Under the heading “Results and Conclusions,” however, the paper is incomplete as there are no conclusions. Before considering any conclusions, however, a correction should be made in the paragraph which precedes Table 1 (on page 213) as this will change some of the values in the table. Here, the author refers to “design lengths” which he used for the toriconical head design in his paper. Evidently he is confused between the Code rules for cylindrical shell design and for conical head design.

The cone heads of both test vessels come under the design rule in Paragraph UG-33(f)(2) of the ASME Code. According to this rule, the allowable pressure is practically the same for both cone heads, 5.3 psi. With these values, the correct ratio of collapsing pressure to code allowable pressure is 4.2 for the 120-deg head and 4.5 for the 90-deg head.

Conclusions from the tests should include a comparison of the test results with the Code rules. In other words, what have the tests indicated? What have we learned? It seems that they show that vessels designed in accordance with the ASME Code rules develop the strengths expected of them. This, at least, should be stated as a conclusion.

H. W. Marsh

I wish to commend Professor Jones on his very excellent paper. It is a worthy addition to the many fine treatises resulting from previous projects sponsored by the Pressure Vessel Research Committee of the Welding Research Council.

My comments are directed primarily toward the calculations of allowable pressures in accordance with Code rules, the apparent factors against collapse, and some conclusions which may be drawn from them. Some variation in the calculated values may be expected because of the omission of certain dimensions. I assumed the head nozzle to be 12.75 in. OD.

I will start with Vessel No. 1 (Fig. 3). Calculations are based upon the following dimensions:

1. T.L. to Stiffener (F&D Head)—24.31 in.
2. T.L. to Stiffener (Toriconical Head)—30.81 in.
3. Depth of Toriconical Head (to base of nozzle)—16.06 in.
4. Depth of F&D Head—10.31 in.

Paragraph UG-33 (f)(2) of the Code states that a toriconical head with an apex angle of from 45 to 120 degrees is to be considered as an equivalent cylinder having a Do equal to the largest inside diameter of the cone—normally considered to be equal to the inside diameter of the vessel for a 6 per cent knuckle head, and with a length equal to the lesser of either: (a) the center-to-center dimension between stiffeners measured parallel to the axis; or (b) a length equal to the diameter (L/D = 1.0). The intent of Case (a) was to cover the situation where stiffeners were welded to the conical head. Thus Case (b) would govern under the Code rule in this instance.

Case (b)

\[ L/D = 1.0; \quad D/t = 447; \quad B = 2200 \]
Max. allowable pressure = 4.9 psig
Theor. buckling pressure = 19.6 psig

For the cylindrical shell adjacent to this head:

\[ L = 30.81 \text{ in.}; \quad D/t = 507; \quad B = 3200 \]
Max. allowable pressure = 6.3 psig
Theor. buckling pressure = 23.2 psig

Case (b) does not appear to be really representative of the existing conditions in this instance. If it is arbitrarily assumed that the nozzle is the equivalent of a stiffener and that Case (a) be applied by using the dimension from the base of the nozzle to the first shell stiffener as the design length, the calculated pressure would be as follows:

\[ L = 30.81 \text{ in.}; \quad D/t = 507; \quad B = 3200 \]
Max. allowable pressure = 6.3 psig
Theor. buckling pressure = 23.2 psig

On this basis the governing factor would be the shell with a calculated allowable pressure of 6.3 psig and a factor against collapse of 3.5 based on the actual collapsing pressure of 22 psig.

The appearance of the collapsed vessel as shown in Fig. 27 would lead me to believe that the failure may have started in the shell. Bending stresses introduced from the toriconical and flat heads may have reduced the collapsing pressure.

No details are given for the reinforced flat head, but it appears that it would have been preferable to have cut the shell at the further stiffener to minimize the effect of the flat head. It would also have been desirable to install viewing ports in the containment vessel with the hope that the mode of initial failure might have been observed.

The design length for the toriconical heads as defined in the paper is incorrect. The definition given is that for the design length of the adjacent shell.

The calculated maximum allowable working pressure for the torispherical (F&D) head, based on a thickness of 0.110 in., should be 5.4 psi rather than 5.9 psi. The design pressure for the adjacent shell is 5.5 psi so there seems to be little likelihood that the failure would have started in the shell.

There appears to be 6 lobes in the contour plot in Fig. 12, but in Fig. 14 it looks as if an 8 lobe pattern was trying to form. This apparently snapped back into 6 lobes prior to collapse. The deformation of the collapsed head is such that it is not surprising that the failure carried over into the adjacent shell section even though the latter was theoretically considerably more stable.

For Vessel No. 2 (Fig. 4) the calculations are based on the following dimensions:

1. T.L. to Stiffener (Ellip. Hd.)—25.28 in.
2. T.L. to Stiffener (Toriconical Head)—31.25 in.
3. Depth of Toriconical Head (to base of nozzle)—25.58 in.

Toriconical Head for assumed Case (a) as in Vessel No. 1
\[ L = 56.83 \text{ in.}; \quad L/D = 0.933; \quad D/t = 451; \quad B = 2400 \]
Maximum allowable pressure = 5.43

For Case (b)
\[ L/D = 1.0; \quad D/t = 451; \quad B = 2200 \]
Maximum allowable pressure = 4.0 psig

For the cylindrical shell adjacent to this head:
\[ L = 39.78 \text{ in.}; \quad L/D = 0.653; \quad D/t = 506; \quad B = 2850 \]
Maximum allowable pressure = 5.62 psig

Although Case (a) would theoretically govern over the shell design pressure, the values of 5.43 and 5.62 psig, respectively, are close enough that there is still considerable doubt as to whether the failure originated in the head or in the shell. If in the shell, the F.C. would be 4.3. The other comments on vessel No. 1 are equally applicable here.

The calculated pressure for the elliptical head, based on a thickness of 0.088 in. is 4.67 psig. The allowable pressure for the
adjacent shell section is 7.9 psig. The shell did not buckle in this case. This is probably due to the less severe deformation and creasing of the elliptical head as compared to the F&D head.

If there is a sufficient area in the head having a reduced thickness to accommodate a peak—about 10–12 per cent of the radius—the head might be expected to fail at a reduced pressure. The thickness traverises shown in Figs. 7 and 8 would appear to justify a reduction of about 5 per cent in the thicknesses used in calculating the theoretical buckling pressure.

In general, the test results reported in the paper appear to prove a reasonable confirmation of the basis for the Code rules with the possible exception of the rules for the design of torispherical heads and reducer sections.

However, the work covered by this paper is barely a beginning with regard to the acquisition of sufficient data upon which to base a more rigorous analysis of cones and conical sections subjected to external pressure. Mention should be made of additional tests which have been conducted by the Nvy Department at the David Taylor Model Basin.

R. G. Sturm

The writer believes that the results of this paper are best interpreted in the light of the general objectives, as he understands them, of the Design Division of P.V.R.C. in planning the test program of which these tests are a part. The need for experimental evidence contributing to the answers to three basic questions had become apparent in the development of the ASME Code for Unfired Pressure Vessels. These questions are:

1. Does the knuckle region of a vessel with a toriconical or torispherical head support the cylindrical shell as a stiffener? The corollary to this question may be stated as "Is the effective length of the cylindrical shell determined as the distance from the nearest stiffener to the center of the knuckle region or should the effective length include a part of the head?"

The Code rules, in Paragraph VG 33 of the ASME Code for Unfired Pressure Vessels, include a part of the head depth in the effective length of cylinder.

2. Does a torispherical head act as a part of a sphere and buckle as a sphere buckles?

3. Does an ellipsoidal head act as a part of a sphere and buckle as a sphere buckles?

The answers to these questions, based on observations of actual vessels and theoretical considerations, has determined the type of formulas currently used to compute the buckling pressure and consequently the safe working pressure for vessels such as those tested. This paper gives data which definitely support the design formulas and rules as given in the ASME Code.

It is unfortunate that limitation on the length of this paper has not permitted the inclusion of the contour plots around the knuckle regions of the toriconical heads (the writer has been privileged to see such plots). The author can confirm the conclusion that the data show that the knuckle region deflected both radially and rotationally as the external pressure was applied. Such behavior indicates that the shell was not fully supported at the juncture and that collapse occurred as a combined action of both head and shell.

The writer understands that the circularity of each vessel was determined from a number of diameter measurements and the initial out-of-roundness of each vessel tested was within the Code tolerances. The measured thicknesses show variations which are common in commercial construction. Therefore, these test vessels are considered to be representative of good commercial construction.

The values of collapsing pressures for the test vessels with toriconical heads indicates that the Code rules are safe and not wasteful for commercial vessels of these proportions. The collapse of the head, as indicated as an alternate in the Code, was not confirmed.

The contour plots for the torispherical head indicate a buckle pattern not inconsistent with the pattern indicated by the theoretical analysis of spheres by "Karman and Tsien." The development of such a pattern definitely indicates buckling of the head as a sphere and justifies the rules for design of the heads as parts of spheres. The test value indicates that for torispherical heads, the Code rules are safe but not wasteful.

The collapse of the ellipsoidal head progressed continuously inward over all of the central part of the head having the larger spherical radius. A slight outward deflection around the outer toroidal region was also continuous. These deflections continued until the head finally snapped through. This action is not inconsistent with the behavior predicted by theoretical analyses both as to character and extent. Again, the test value indicates that, for the design of ellipsoidal heads, the Code rules are safe and not wasteful.

It is gratifying and reassuring to find that the nature of failure of each of the test vessels is consistent with the diagnosis upon which the design rules given in the ASME Code have been based. The collapse values indicate adequate factors of safety over the allowable values computed in accordance with the Code.

Leonard P. Zick

The author is to be congratulated for presenting his experimental results in the complete manner he has. Designers can make their own calculations and most will take issue with the theoretical ASME calculations used in his comparison in Table 1. Actually the ASME Code has two rules for pressure vessels designed to resist external pressure, either of which might govern a particular design. Collapse of the various head forms is covered in Par. UG-33 and collapse of the shell plus part of the head is covered in Par. UG-28(c). Evaluation of the latter condition is further complicated in the case of the test vessels because the shell and head are of different thicknesses. The author apparently used the head thickness and Par. UG-28(c) in arriving at 8.5 psi for the 120-deg toriconical head and 8.4 psi for the

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Table 2

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Calculated by</th>
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<th>Calculated by</th>
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<tr>
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<td>5.7 psi</td>
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<tr>
<td>P</td>
<td>7.4 psi</td>
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* President, Sturm-Krouse, Inc., Auburn, Ala. Mem. ASME.
90-deg toriconical. Since the cylindrical shell is thinner than the conical head and since the cylindrical shell represents a major portion of the length, the thickness of the cylindrical shell should be used.

Par. UG-28(c) does not govern in the case of the elliptical head plus adjacent shell nor did the shell buckle. The torispherical head plus shell figures not to fail according to Par. UG-28(c). However, both head and shell buckled as in the test of the toriconical heads. In applying Par. UG-28 the shell thickness (or a weighted average thickness) should again be used even though the shell is thicker than the heads.

Table 2 summarizes the allowable pressures by both paragraphs for the four heads. The heads are listed in order of their collapsing pressure. Note that Par. UG-28(c) predicts the reverse order for the three head and shell combinations where the shell failed. This paragraph also predicts the shell and torispherical head, which failed, to be stronger than the shell and elliptical head.

The toriconical heads are assumed to be governed by Par. UG-33(f)(2). However, the present Code wording leaves out an apex angle of exactly 120 deg and does not specifically cover a cone head with a knuckle.

This paragraph also says that the effective length is equal to the cylindrical diameter regardless of the apex angle between 120 and 45 deg. In view of the buckled conical heads it would appear that the nozzle acts like a stiffener in preventing buckling of the center of the head. As an alternate to the arbitrary \( L = D \), suggested calculations are also shown in Table 2 using:

\[
L = \text{the slant length of the cone element measured from the stiff opening to the intersection of the projection of the cone element and the cylindrical shell}
\]

and

\[
D = \text{twice the conical radius that is used to compute the internal pressure of a cone}
\]

It would be of interest to know the pattern of the stiffeners on the flat head. Is it possible that these stiffeners stiffened the flange in such a way as to cause the eight lobe pattern of failure in the three cylindrical shells?

**Author's Closure**

The author wishes to thank Messrs. Bergman, Hartman, Marsh, Sturm, and Zick for their discussions of the paper.

Mr. Bergman, Mr. Hartman, and Mr. Marsh made reference to the fact that the design lengths used by the author in calculating the ASME code allowable pressures for the toriconical heads were design lengths for cylindrical shells. It will be recalled that each toriconical head was tested as a cylinder-toriconical head unit. It was also observed from Figs. 27 and 37 that the cylindrical shells appeared to suffer more damage than the heads; hence the shells, rather than the heads, apparently governed the actual collapse pressure of the head-shell unit. Therefore in determining the code allowable pressures for the head-shell units, the author treated the units as equivalent cylinders according to Paragraph UG-28(C) of the 1959 ASME Boiler and Pressure Vessel Code for Unfired Pressure Vessels.

It should be pointed out, as Mr. Zick has observed, that the Code does not include the specific cases of the two toriconical head units considered in this investigation. The Code wording leaves out an apex angle of exactly 120 degrees and it does not specifically cover the case of a cone head with a knuckle.

Mr. Zick indicated that it would be of interest to know the pattern of stiffeners on the flat head. There were eight stiffeners cut from a one-half-inch flat plate. These stiffeners were welded to the plate in a radial pattern. They were equally spaced, and each stiffener extended from the center of the plate to within approximately three inches of the edge of the plate. Each stiffener was approximately six inches high at the plate center and approximately two inches high at the outer edge.

Observation of the collapsed toriconical head-shell units revealed the fact that the outer edge of each of several stiffeners tore through the cylindrical shells. This enabled one to determine the position of the stiffeners relative to the head-shell unit. An investigation of both collapsed toriconical head-shell units revealed that the stiffeners were located between the adjacent lobes of the cylindrical shells of the units.

Mr. Hartman indicated that "...the paper is incomplete as there are no conclusions." In the "Results and Conclusions" section of the paper, the author indicated such conclusions as: the stresses in the torispherical-head weld junction region were generally higher than for the similar regions on the other three vessel heads; the maximum circumferential stress in the torispherical head at the collapse pressure exceeded the average yield point of the steel (figures were given to show that the yield point was exceeded by more than forty per cent); and Table 1 reflected conclusions concerning the observed collapse pressure... Code allowable pressure comparisons.

Mr. Hartman pointed out that, "Conclusions from the tests should include a comparison of the test results with the Code rules." It is the author's opinion that this was done in summary form in Table 1.